

**Optical pulling forces in hyperbolic metamaterials**Alexander S. Shalin,<sup>1,2,3,\*</sup> Sergey V. Sukhov,<sup>1,2,4</sup> Andrey A. Bogdanov,<sup>1,5,6</sup> Pavel A. Belov,<sup>1</sup> and Pavel Ginzburg<sup>7,†</sup><sup>1</sup>*ITMO University, St. Petersburg 197101, Russia*<sup>2</sup>*Kotel'nikov Institute of Radio Engineering and Electronics of Russian Academy of Sciences S (Ulyanovsk branch), Ulyanovsk 432011, Russia*<sup>3</sup>*Ulyanovsk State University, Ulyanovsk 432017, Russia*<sup>4</sup>*CREOL, The College of Optics and Photonics, University of Central Florida, 4000 Central Florida Blvd., Orlando, Florida 32816, USA*<sup>5</sup>*Ioffe Institute, St. Petersburg 197101, Russia*<sup>6</sup>*Peter the Great St. Petersburg Polytechnic University, St. Petersburg 195251, Russia*<sup>7</sup>*School of Electrical Engineering, Tel Aviv University, Tel Aviv 69978, Israel*

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Control over mechanical motion of nanoscale particles is a valuable functionality desired in a variety of multidisciplinary applications, e.g., biophysics, and it is usually achieved by employing optical forces. Hyperbolic metamaterials enable tailoring and enhancing electromagnetic scattering and, as the result, provide a platform for a new type of optical manipulation. Here optical pulling forces acting on a small particle placed inside a hyperbolic metamaterial slab were predicted and analyzed. In order to attract particles to a light source, highly confined extraordinary modes of hyperbolic metamaterial were excited via scattering from an imperfection situated at the slab's interface. This type of structured illumination together with remarkable scattering properties, inspired by the hyperbolic dispersion in the metamaterial, creates optical attraction. Forces acting on high-, low-index dielectric, and gold particles were investigated and it was shown that the pulling effect emerges in all of the cases. The ability to control mechanical motion at nanoscale using auxiliary photonic structures paves the way for investigation of various phenomena, e.g., biochemical reactions, molecular dynamics, and more.

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**I. INTRODUCTION**

Electromagnetic radiation could transfer its inherent mechanical momentum to different structures via scattering and absorption. A self-consistent electromagnetic field interacts with induced polarization charges and, as a result, the macroscopic Newtonian force emerges. A successful optical trapping experiment with the focused laser beam (optical tweezers [1]) started the era of optical manipulation [2]. Since then this concept was employed in various fields of science and engineering, among them are laser cooling [3], particles' sorting [4], studies of artificial crystals formed by optical lattices [5], quantum computing [6], solar sails [7], studies of conformations events at molecular level [8], optomechanical light modulators [9], and many other areas.

One of the primarily goals of optical manipulation research is to reduce the overall power of a trapping beam and to achieve stiff and highly localized potential profiles of light. Apart from the undesirable environmental temperature effects, high light intensities could be harmful for trapped objects, especially for the biological specimens (e.g., Ref. [10]). Complex shaping of beams could improve the quality of trapping (e.g., Ref. [11]); however, being based on diffractive optical elements, their spatial resolution is limited by the classical diffraction. A promising approach for optical tweezers with subdiffraction light confinement is to utilize plasmonic nanostructures. Noble metals with negative permittivity at optical and infrared spectral range can support highly localized surface plasmon modes exhibiting strong field gradients. Plasmonic tweezers have advantages over conventional diffractive optics elements

in terms of improved spatial localization of traps and overall reduction of required light intensity [12]. Optical manipulation with auxiliary nanostructures is a very promising and already proven concept [13,14]. Nevertheless, most of the considered nanostructures provide static localized light spots, making dynamical control over the trapped objects more complicated. In this sense, the ability to create highly localized waves with high spatial resolution on demand could be very beneficial. Metamaterials could serve as a flexible platform for controlling light propagation on nanoscale and tailoring scattering properties of embedded objects [15]. Hyperbolic metamaterials [16] supporting strongly confined high density of states modes could be beneficial for those purposes. Those remarkable properties emerge due to a hyperbolic type of dispersion in the artificially created media having opposite signs of effective permittivities along principal axes of the crystal. Hyperbolic metamaterials could be fabricated as arrays of vertically aligned nanorods or periodic metal-dielectric layers [17–19]. Relying on the above, the investigation of optical forces in hyperbolic metamaterials is of considerable interest and could lead to new remarkable phenomena, in particular, optical attraction.

The phenomenon of optical attraction toward illumination sources gained quite considerable attention [20], as it provides additional flexibility for optical manipulation. Most commonly, this effect is achieved by either configuring the spatial shape of the incident beam (hollow beams, Bessel beams, and more [20–23]) or by shaping the geometry and material composition of a scatterer [20]. In the latter case the requirement of forward scattering predomination over the rest of the interaction mechanisms should be fulfilled. In other cases, the effect of optical attraction can be achieved by designing properties of surrounding medium [24–28]. In particular, this approach was proposed for negative index

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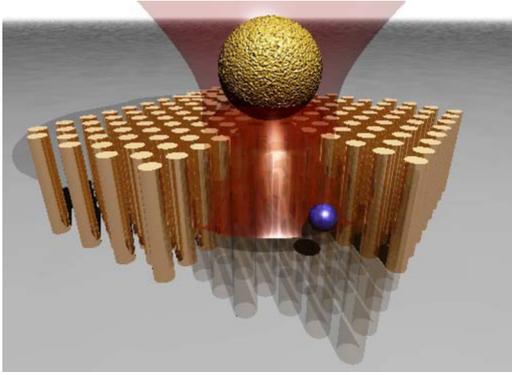


FIG. 1. (Color online) Schematics of the general concept. Extraordinary waves in a metamaterial excited via scattering on an obstacle. Small particle, situated inside the material, experiences optical forces. Some of the nanorods appear transparent for representing the inner geometry.

media, which is still challenging to obtain in the optical regime. Here the concept of auxiliary nanostructuring will be extended to the case of routinely fabricated hyperbolic metamaterial serving as embedding medium for tailoring both illumination and scattering properties of manipulated objects.

Here we investigate optical forces acting on nanoparticles placed inside a hyperbolic metamaterial slab (Fig. 1). Highly confined extraordinary modes of the hyperbolic metamaterial are excited via scattering of external illumination on an inhomogeneity (nanosized cylinder) situated on the slab's interface. This scenario comes to emulate various possible experimental arrangements, e.g., excitation through an aperture in opaque mask (static control) or illumination with a near field scanning optical microscope tip (dynamic control with movable tapered fiber). The finite element numerical analysis predicts emergence of optical forces attracting metal (gold), high-, and low- index dielectric nanoparticles to a light source. Optical pulling forces were demonstrated for both polarizations of the incident illumination with respect to the negative component of the metamaterial permittivity tensor. Furthermore, particles were shown to be trapped in the vicinity of the extraordinary wave's source (upper facet of the metamaterial on Fig. 1).

## II. EXCITATION OF HIGHLY CONFINED MODES IN HYPERBOLIC METAMATERIAL VIA SCATTERING

Electromagnetic scattering is the result of reradiation of fields by coherently excited polarization charges in material structures. Scattering cross sections and patterns strongly depend on the properties of the environment around an object, in particular, on available electromagnetic modes in the surrounding, their spatial distribution, and density of states. Hyperbolic metamaterials, both as an embedding or nearby medium, support the modes with a high density of states, in which the radiation could scatter, as was shown in several configurations (e.g., Refs. [29–31]). These modes propagate only in the narrow range of directions forming a characteristic cross-shaped scattering pattern.

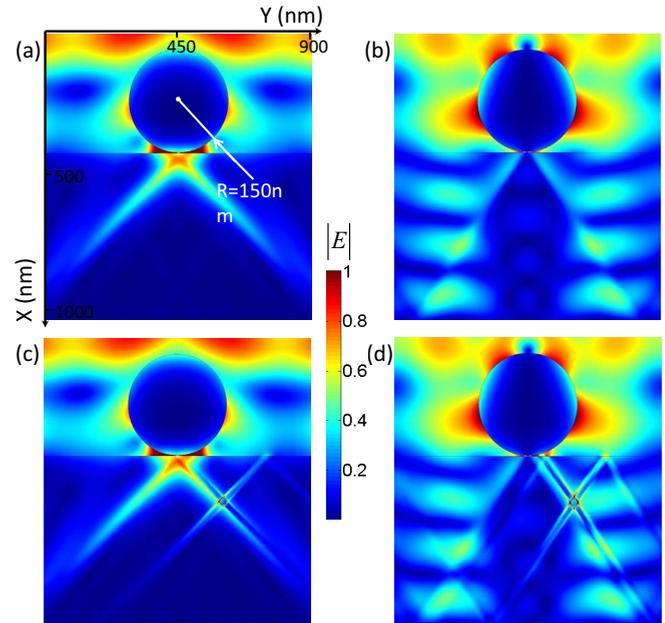


FIG. 2. (Color online) Intensity distribution (a), (b) without and (c),(d) with a gold particle in the metamaterial slab. Effective permittivity tensor: (a),(c) nonpenetrable case  $\epsilon_x = 3 + i0.5$ ,  $\epsilon_y = -3 + i0.5$ ; (b),(d) penetrable case  $\epsilon_x = -6 + i0.5$ ,  $\epsilon_y = 3 + i0.5$ . The incident wave (540 nm wavelength) is polarized along the  $y$  axis.

In order to simplify the numerical analysis, two-dimensional geometry was considered. Therefore, all the following results refer to the linear force density acting on elongated (effectively infinite) objects. A gold cylindrical obstacle with 150 nm radius was situated on top of a homogeneous hyperbolic metamaterial (Fig. 2). Two different sets of material parameters were considered:  $\epsilon_x = 3 + i0.5$ ,  $\epsilon_y = -3 + i0.5$  or  $\epsilon_x = -6 + i0.5$ ,  $\epsilon_y = 3 + i0.5$ , corresponding to achievable realization either with metal layers or nanorods. Homogenized hyperbolic metamaterials with effective dielectric functions corresponding to those structures were analyzed. Homogenization of optical properties of the composite could be performed with various techniques, e.g., for nanorod geometry, the corresponding methods were discussed in Ref. [32]. Metamaterials with small compared to excitation wavelengths unit cells approach performances of the idealized model.

The finite element method [33] was used for the calculation of electromagnetic fields induced by an incident  $y$ -polarized plane wave propagating along the  $x$  axis (Fig. 2). The illumination wavelength is 540 nm; the permittivity of gold is  $\epsilon_{Au} = -5.15 + 2.21i$ . Figures 2(a) and 2(b) show the distribution of electric-field intensity in the metamaterial slab standalone (without introducing manipulated particles inside). Two scenarios of the scattering were considered: (i) wave's polarization coincides with the negative component of the permittivity tensor ( $\epsilon_y = -3 + 0.5i$ ); (ii) wave's polarization coincides with the positive one ( $\epsilon_y = 3 + 0.5i$ ). In the first case [Fig. 2(a)], the illumination is reflected from the surface and the only excitation channel of the field inside the slab is the scattering from the cylinder. This specific property of hyperbolic metamaterials allows getting rid of the background illumination and, as a result, enables gaining sharply defined

optical potentials. A qualitatively different situation is obtained for the second case when the incident wave is polarized along the positive tensor component of the metamaterial [Fig. 2(b)]. In this case, both directly transmitted and scattered waves contribute to the pattern formation inside the metamaterial and highly localized extraordinary modes appear on the plane-wave background [Fig. 2(b)]. In both scenarios, however, the scattered field has a cross-shaped pattern indicating the contribution of extraordinary modes (e.g., Ref. [34]).

At the next stage, a small object is introduced inside the metamaterial slab. A nanoparticle of 10 nm radius, placed in the vicinity of abrupt variations of the intensity, exhibits remarkable rescattering of the background field. Electromagnetic field distribution in the presence of this small gold particle is shown in Figs. 2(c) and 2(d). It could be seen that this small perturbation acts as a secondary source of extraordinary waves in the metamaterial with the characteristic cross-shaped pattern. Reflections of the extraordinary waves from the slab's boundaries are observed; multiple reflections of the same wave are suppressed due to the presence of losses in the structure. The embedding metamaterial changes the free space properties of the particle, leading to both cross-section enhancement and directionality of the scattering. Those two effects are the key for observing unusual properties of optical forces.

### III. OPTICAL FORCES ACTING ON A PARTICLE PLACED IN HYPERBOLIC METAMATERIALS

The general framework for electromagnetic forces calculation is the Maxwell's stress tensor. This procedure requires the knowledge of self-consistent fields on an arbitrary surface  $\Sigma$  enclosing an object [35]:

$$\langle T_{\alpha\beta} \rangle = \frac{1}{2} \text{Re} \left[ \varepsilon_0 E_\alpha E_\beta^* + \frac{1}{\mu_0} B_\alpha B_\beta^* - \frac{1}{2} \left( \varepsilon_0 |\mathbf{E}|^2 + \frac{1}{\mu_0} |\mathbf{B}|^2 \right) \delta_{\alpha\beta} \right], \quad (1)$$

where  $\alpha, \beta = x, y, z$ ,  $\mathbf{E}$  and  $\mathbf{B}$  are the electric field and magnetic induction, and  $\varepsilon_0, \mu_0$  are the permittivity and permeability of vacuum. While optomechanical forces acting on bodies suspended in lossless dispersion free media are well defined, in the case of arbitrary media (lossy and dispersive) they should be treated with extra care. In the case of the metamaterial environment, a thin vacuum shell was imposed around the particle in order to prevent an ambiguity with the definitions (Abraham-Minkowski controversy; e.g., Ref. [36]). This exclusion of the material volume in the vicinity of the optically manipulated object also allows one to phenomenologically account for finite dimensions of the metamaterial unit cell, which has a major impact on the diversity between the homogenized model and realistic implementation of metamaterials. The time-averaged electromagnetic force is given by

$$\langle \mathbf{F} \rangle = \int_{\Sigma} \langle \hat{T}(\mathbf{r}, t) \rangle \mathbf{n}(\mathbf{r}) d\sigma, \quad (2)$$

where  $\mathbf{n}(\mathbf{r})$  is the outward normal to the surface  $\Sigma$ . In our case, the surface  $\Sigma$  corresponds to a 12 nm radius shell comparable to the periodicity of a typical nanorods structure.

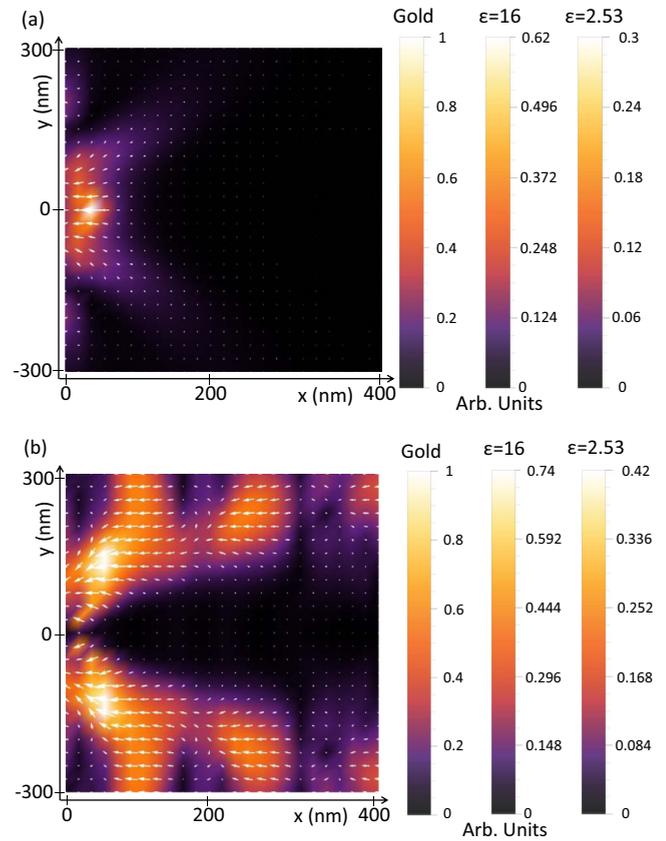


FIG. 3. (Color online) Linear density of optical force (arbitrary units) as a function of particles' coordinates. (a) Metamaterial with  $\varepsilon_x = 3 + i0.5$ ,  $\varepsilon_y = -3 + i0.5$ ; metamaterial with  $\varepsilon_x = -6 + i0.5$ ,  $\varepsilon_y = 3 + i0.5$ . Different color bars correspond to particles made of different materials: gold ( $\varepsilon_{Au} = -5.15 + 2.21i$ ), high index dielectric ( $\varepsilon = 16$ ), and low index dielectric ( $\varepsilon = 2.53$ ).

The numerically calculated optical linear force densities [with the help of Eqs. (1) and (2)] on either gold ( $\varepsilon_{Au} = -5.15 + 2.21i$ ) or dielectric particles with high ( $\varepsilon = 16$ ) and low ( $\varepsilon = 2.53$ ) index for the two considered metamaterials (orientation of the major permittivity tensor axis in accordance to the polarization of the incident wave) are shown in Fig. 3. The magnitude of the force density is shown with color, while the direction of the force is represented by white arrows. All three cases show remarkable similarity in spatial dependence apart from the magnitude of the force. Three different color bars correspond to different materials of the particle. The highest linear force density is obtained for the gold particle, while the low index one shows the smallest values. This scaling property of forces is quite remarkable, since the self-consistent electromagnetic fields do depend on the permittivity of a scatterer. In principle, this type of optical forces problem does not always scaling with the particle's polarizability. The estimated ratio of maximal forces is  $|F_{gold}^{max}|/|F_{n=1.59}^{max}| \sim 3$  and  $|F_{gold}^{max}|/|F_{n=4}^{max}| \sim 1.5$ , for both orientations of the metamaterial with respect to the polarization of the incident wave. Remarkably similar ratios could be indeed obtained by comparing the particle's polarizabilities in vacuum  $\{\alpha \sim [(\varepsilon_{particle} - 1)/(\varepsilon_{particle} + 1)]\}$ . For better understanding of this scaling property, the time-averaged force in

the simplified conditions of a monochromatic illumination and a small scattering dipolar particle in vacuum could be written as [35]

$$\langle \mathbf{F} \rangle = \frac{\alpha'}{4} \nabla \langle |\mathbf{E}_{\text{inc}}|^2 \rangle + \frac{\alpha''}{2} \langle |\mathbf{E}_{\text{inc}}|^2 \rangle \nabla \phi_{\text{inc}}, \quad (3)$$

where  $\alpha = \alpha' + i\alpha''$  is the complex particle's polarizability,  $\phi_{\text{inc}}$  is the phase of the incident field, and  $\mathbf{E}_{\text{inc}}$  is the incident electrical field at the location of the particle. Note that the incident field is used for calculation of optical forces within the dipolar approximation [Eq. (3)], while the complete self-consistent electromagnetic field enters the Maxwell's stress tensor in Eq. (1). The first term in Eq. (3) characterizes gradient optical forces, while the second one represents the radiation pressure. The estimation of polarizability ratios for the particles gives the following values:  $\alpha'_{\text{gold}}/\alpha'_{n=4} = 1.6$  and  $\alpha'_{\text{gold}}/\alpha'_{n=1.59} = 3.2$  with very good agreement with the numerically obtained force ratios. It means that the spatial structure of the scattered field in hyperbolic metamaterial weakly depends on the material of the scatters, which only predefines the scattered intensity. That's an additional remarkable property of hyperbolic metamaterials and is the result of high density of states available for scattering. Those force ratios also allow concluding that optical pulling occurs due to the gradient forces (depending on the real part of polarizabilities), while in the case of the golden particle, the radiation pressure slightly weakens the attraction.

All the considered types of particles move to the upper interface of the metamaterial slab, i.e., against the light propagation direction, and that is the essential difference from the classical optical tweezers. It means that the field in the depolarization volume (the air shell imposed around the particle) is concentrated in the part closer to the light source. If the incident wave is polarized along the positive permittivity component (penetrable case), the force map [Fig. 3(b)] has a more complex structure than in the nonpenetrable scenario [Fig. 3(a)]. Nevertheless, in both cases the major effect of optical attraction is preserved.

It is worth noting that intensity gradients, valuable for optical forces, could be achieved by introducing material losses or strong scattering channels. For example, optical attraction could emerge in dense colloidal systems where strong scattering produces sufficient intensity gradients [37]. In order to resolve the nature of the attraction force in the scenario considered here, an additional set of numerical experiments was performed. Values of optical forces emerging in structures with  $\varepsilon_y = -3 + 0.5i$  and  $\varepsilon_y = -3 + 1i$  were compared with the previous set of results. The incident illumination amplitude was adjusted in the way the intensities on the particle at each case are equal. The position of the particle was chosen to be at the center of the beam. The resulting force was estimated and it showed a 10% drop with increasing the nominal material losses by a factor of 2. It means that in the considered relatively complex geometry the losses are not directly related to the field gradient, but rather contribute to complex electromagnetic dynamics within the structure. Under certain circumstances, however, the approach of enlarging gradient forces on the expense of optical losses increase might work, but requires detailed analysis of each individual structure and realization.

Extraordinary waves in the metamaterial create the shadow area below the excitation scatterer. This property is the result of the modified diffraction phenomena owing to the hyperbolic dispersion regime. The same property is in charge for the scattering on a small optically manipulated particle and, as a result, the unusual optical force emerges. Highly confined modes also cause the resulting high stiffness of the optical trapping region.

#### IV. OUTLOOK AND CONCLUSION

Optical manipulation and control of nano- and microscale particles is paramount for development of the next generation of lab-on-a-chip optofluidic and sensing applications as well as for advanced nanooptomechanical systems for photonics. Nowadays, considerable attention is devoted to the investigations of novel approaches for optomechanical control of nanostructures' transport.

Here we analyzed optical forces acting on nanoparticles placed inside homogeneous hyperbolic metamaterial slabs. The finite dimension of the metamaterial unit cell was taken into account via an introduction of depolarization volume around the particles, while the electromagnetic properties of the hosting material were homogenized. Rigorous numerical analysis shows that optical forces in a hyperbolic medium can pull the particle to the metamaterial interface towards the light source. This phenomenon is explained by remarkable diffraction properties of embedding media, supporting strongly localized high density of states extraordinary modes with steep intensity gradients. It should be noted, however, that the fine structure of optical forces could be significantly influenced by a realization of the metamaterial's unit cell, which could enforce certain constraints. For example, near field interactions between a particle and the nearest unit cell of the composite strongly depends on material composition and the size of the cell. Furthermore, both nanorods' and layers' realizations of the metamaterials pose certain physical limitations on available particle positions, e.g., layers will prohibit any motion across the layers. Metamaterial realization in the form of magnetized plasma [38] will bring into consideration another span of effects, but, for example, the size of the unit cell will be less relevant here. Consequently, the choice of depolarization volume, nevertheless, was aimed to bring the general scope of possible phenomena into consideration.

On the contrary to other existing methods for achieving optical attraction, the proposed scheme does not require shaping of beams or particles' geometries or materials. Performed numerical analysis shows that the optical pulling force is observed for both polarizations of the incident light either along positive or negative permittivity components of the metamaterial. It is worth briefly noting that optical attraction with negative index metamaterials was predicted even at the seminal work of Veselago [24]. The discussion about existence of these forces still continues to present day [25,26]. However, practical realizations of those materials in the optical domain are still challenging, giving advantage to large scale hyperbolic metamaterials. Additional discussion on potential differences between two-dimensional (2D) and 3D models is of potential interest too. The 2D model considered here represents a scenario of elongated objects. The results were obtained for the

linear force density and could be directly applied on the relevant cases. In 2D geometry, the highly confined extraordinary waves in the metamaterial are almost nondiffracting, as losses have minor impact on this behavior. In this case, the only source of the beam attenuation (intensity gradient) is optical losses. On the other hand, in the 3D case the light will propagate along a conical surface (the opening angle is solely defined by the ratio of ordinary and extraordinary components of the permittivity). As the perimeter of the cone basis grows proportionally to the distance from its apex, the intensity will show  $1/\text{distance}$  decay law into the medium (in the 2D case this channel is absent). The radiation pressure, hence, will drop linearly with the distance. However, the gradient force is proportional to the gradient of intensity ( $1/\text{distance}^2$ ) and drops much faster away from the interface. That suggests a presence of a point (close to the surface), where two types of forces could balance each other. Thus, the natural gradient of intensity still creates attractive forces in the vicinity of the interface. It should be noted, however, that optical losses cause exponential decay of the intensity, which in the majority of cases will define the field profile and, as a result, the value and directions of optical forces.

In summary, the proposed model of optical attraction in hyperbolic metamaterial provides alternate approaches to the field of optical manipulation and makes the general concept of metamaterials mediating optomechanical interactions to be a promising platform for advanced control. For example, the presence of controllable shadow areas could enable optical triggering of colloidal separations and mixtures.

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